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## Probabilistic Economic Emission Dispatch Optimization of Multi-Sources Power System

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### Abstract

The interest on renewable energy resources is growing and the study of different integration aspects of these resources becomes very important to overcome problems caused by their variability or uncertainty. This paper treats the economic environmental power dispatch as a probabilistic multiobjective problem. The operation cost and green house gas emission functions are considered as the sum of deterministic part and probabilistic one. First, the problem is solved based on expected values of generated wind power then, using the cumulative density function (CDF) of each renewable energy source (RES), the CDF of the required reserve to compensate the RESs variability in order to keep the power balance. Then, respecting to the reserve contribution of each thermal generator, the probabilistic part of the global generation cost as well as its CDF are developed. Finally, the proposed approach is applied to solve the active power dispatch problem of IEEE 30-bus test system in two cases with and without RESs. The simulation results show that this method allows to get the complete information about the cumulative distribution function of the actual global cost of the system operation.

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### 1. Introduction

With the growing interest to use renewable and sustainable new energy sources (RESs) for economic and/or environmental reasons, systems power operators have to start changing their power management policies because of the changing conducted by the intermittent RESs generation. In [1], a probabilistic approach based on the convolution technique to assess the long-term performance of a hybrid solarwind power system is developed in order to deal with the RES variability in the economic dispatch. Other studies have been done to reach a great power management in microgrids such as the approach developed in [2] which proposes a dynamic assignment of renewable energy tokens algorithm for collaborative microgrids based on the load management side and allowing to keep the power balance.

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Besides, the paper [3] proposes non-uniform hierarchical 16-QAM to provide a reliable data transmission over wireless links to achieve an efficient information exchange between the participants in such collaborative system.

This paper proposes a resolution approach for the economic dispatch problem (EED) of a power system integrating RESs. Both the cost and the greenhouse gas emission of the system operation to minimize as a multiobjective optimization problem. Recently, the use of evolutionary algorithms is increasing due to their abilities to resolve complex problems especially in electrical field such solving problems of active and reactive power dispatch problems [4,5]. This work uses fast and elitist multi-objective genetic algorithm (NSGA-II) to optimize the EED of power system in terms of load supplying and contribution of renewable sources in production power. Besides, a probabilistic study of the required reserve is done in order to give the cumulative distribution function (CDF) of the global operation cost. The reminder of this paper is structured as follows. Section II presents a probabilistic modeling of RESs, Section III develops the problem formulation while Section IV details the used optimization approach. Then, Section IV presents results discussion are conducted in Section V and finally, section V concludes this work.

## 2. Probabilistic power modeling of renewable energy sources

There are various models that express mathematically the electrical power produced by renewable technologies using deterministic or probabilistic approaches [6,7].

### 2.1. Probabilistic modeling of PV cell power

The energy produced by a photovoltaic (PV) generator is estimated based on manufacturer data as well as climate data (radiation and temperature). The output power of the PV generator can be calculated by [8]

$$P_{PV} = rA\eta \quad (1)$$

with

$$\eta = \eta_{ref}(1 - \gamma(T - T_{ref})) \quad (2)$$

where  $r$  is the solar irradiance;  $A$  is the total area of the PV module;  $\eta$  is the PV generation efficiency. On the other hand,  $\eta$  varies with the cell temperature  $T$ , where  $\eta_{ref}$  is the reference efficiency of the photovoltaic generator,  $\gamma$  is the temperature coefficient of short-current [K] and  $T_{ref}$  is the reference cell temperature [K]. The solar irradiance  $r$  can be described reasonably by a beta distribution [9]

$$f_r(r) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \left(\frac{r}{r_{max}}\right)^{a-1} \left(1 - \frac{r}{r_{max}}\right)^{b-1} \quad (3)$$

with

$$a = \mu \left[ \frac{\mu(1-\mu)}{\sigma^2 - 1} \right] \quad (4)$$

$$b = (1-\mu) \left[ \frac{\mu(1-\mu)}{\sigma^2 - 1} \right] \quad (5)$$

where  $r_{max}$  is maximum solar irradiance. In this paper, it is assumed that the PV cell temperature forecasts are without errors. Then the PDF of PV cell power  $P_{PV}$  is given by

$$f_{PV}(P_{PV}) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \left(\frac{P_{PV}}{P_{PV}^{max}}\right)^{a-1} \left(1 - \frac{P_{PV}}{P_{PV}^{max}}\right)^{b-1} \frac{1}{A\eta} \quad (6)$$

where  $P_{PV}^{max}$  is the maximum generated power. Then, the expected values and the cumulative distribution function (CDF) of PV generation are expressed in Eq. 7 and Eq. 8.

$$E(P_{PV}) = \int_{-\infty}^{+\infty} P_{PV} f_{PV}(P_{PV}) dP_{PV} \quad (7)$$

$$CDF(P_{PV}) = \int_{-\infty}^{P_{PV}} f_{PV}(x)dx \tag{8}$$

2.2. Probabilistic modeling of wind power

The output power of a wind turbine varies at different wind speeds and accordingly to the power curve given by the manufacturer. Indeed, the power output of wind turbine can be approximated by [8,10],

$$P_w(v) = \begin{cases} 0 & v < v_c, v > v_f \\ p_r \frac{v-v_c}{v_r-v_c} & v_c \leq v \leq v_r \\ p_r & v_r \leq v \leq v_f \end{cases} \tag{9}$$

where  $p_r$  is the rated electrical power,  $v_c$  is the cut-in wind speed at which the turbine first starts to rotate and generate power,  $v_f$  the Cut-off wind speed which is the breaking system employed to avoid damage to the rotor and  $v_r$  the rated wind speed [m/s] at which the power output reaches the best operating at  $p_r$ .

The wind speed is a random variable which mostly approximated by Weibull distribution [1].

$$f(V) = \left(\frac{k}{c}\right) \left(\frac{V}{c}\right)^{k-1} \exp\left(-\left(\frac{V}{c}\right)^k\right) \tag{10}$$

where  $c$  is a scale parameter and  $k$  is a shape parameter.

The wind power PDF is deduced from Eq.9 and Eq.10 and since the function of wind power in terms of wind speed variable is strictly increasing, the PDF of  $P_w$  can be expressed by

$$f_w(P_w) = \begin{cases} 1 - \exp\left(-\left(\frac{V_c}{c}\right)^k\right) + \exp\left(-\left(\frac{V_f}{c}\right)^k\right) & P_w = 0 \\ \left(\frac{k}{c}\right) \left(\frac{V_c + (V_r - V_c)P/P_r}{c}\right)^{k-1} \exp\left(-\left(\frac{V_c + (V_r - V_c)P/P_r}{c}\right)^k\right) \frac{(V_r - V_c)}{P_r} & 0 \leq P_w \leq P_r \\ \exp\left(-\left(\frac{V_r}{c}\right)^k\right) - \exp\left(-\left(\frac{V_f}{c}\right)^k\right) & P_w = P_r \end{cases} \tag{11}$$

Then, the expected values and the cumulative distribution function (CDF) of wind generator are expressed in Eq. 12 and Eq. 13.

$$E(P_w) = \int_{-\infty}^{+\infty} P_w f_w(P_w) dP_w \tag{12}$$

$$CDF(P_w) = \int_{-\infty}^{P_w} f_w(x) dx = 1 - \exp\left(-\left(\frac{V_c + (V_r - V_c)P/P_r}{c}\right)^k\right) \tag{13}$$

3. Probabilistic economic emission dispatch optimization

This paper considers the economic emission dispatch (EED) problem as the combination of two subproblems. The first one is a multiobjective optimization of fuel cost and greenhouse gases emission of thermal units (TU) which accounts the wind and PV generations by their expected values. While the second subproblem considers the cumulative distribution functions (CDF) of either wind and PV power in order to compute the total necessary reserve. In fact, this work takes into account only the required reserve for compensating the disparity between actual renewable generated power and expected values and considers that reserve is guaranteed by the system TUs and the power contributing in reserve of each one is depending on the the shape slope of the spinning reserve cost.

Using the expected values of renewable sources, the residual load power  $P_D^r$  can be expressed by,

$$P_D^r = P_D - \sum_{i=1}^{N_w} E(P_{Wi}) - \sum_{i=1}^{N_{PV}} E(P_{PVi}) \tag{14}$$

where  $P_D$  is the active power demand. Then, this paper considers the EED problem solution as a multiobjective problem of cost and emission objective functions to satisfy the residual demand. The classical economic dispatch problem of finding the optimal combination of power generation, that minimizes the total fuel cost, while satisfying required demand at each bus [11], is formulated as,

$$f_1 = \sum_{i=1}^{N_G} (a_i + b_i P_{Gi} + c_i P_{Gi}^2), \tag{15}$$

where  $a_i, b_i$  and  $c_i$  are the fuel cost coefficients of generator  $i$ ,  $P_{Gi}$  is the power produced per unit (p.u) by generator  $i$  and  $N_G$  is the number of generators. The amount of greenhouse gas emissions is given as the sum of a quadratic and exponential functions of each generator [11] and is given by,

$$f_2 = \sum_{i=1}^{m_g} (10^{-2}(\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \psi \exp(\lambda_i P_{Gi})), \tag{16}$$

where  $\alpha_i, \beta_i, \gamma_i$  and  $\lambda_i$  are the emission coefficients of generator  $i$ .

Equality constraint: Such constraints presents the active power balance of the whole electrical network. The power losses are neglected in this work the this constraint is formulated by,

$$\sum_{i=1}^{N_G} P_{Gi} - P_D^r = 0, \tag{17}$$

where  $P_D$  and  $P_L$  are, respectively, the active power demand and the active power losses.

*Inequality constraints:* Each voltage and active power  $P_{gi}$  is restricted by an upper and a lower limits, and expressed by,

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \tag{18}$$

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}. \tag{19}$$

Since the generated power of TUs are obtained, the total required reserve can be given as

$$TR(P_{W_1}, \dots, P_{W_{N_W}}, P_{PV_1}, \dots, P_{PV_{N_{PV}}}) = \sum_{i=1}^{N_G} P_{Gi} + \sum_{i=1}^{N_W} P_{W_i} + \sum_{i=1}^{N_{PV}} P_{PV_i} - P_D \tag{20}$$

We note that the expected value of the total required reserve is a null value and its PDF is equal to the convolving product of all RES PDFs. In the case of two generators wind and photovoltaic, the TR CDF is given by

$$CDF(TR) = \int_{-\infty}^{+\infty} F_W(TR - P_{PV}) f_{PV}(P_{PV}) dP_{PV} \tag{21}$$

Finally, the reserve cost is added to the solutions values of  $f_1$ . The cost contributing reserve power  $P_{SRi}$  can be expressed by [12],

$$C_{SRi}(P_{SRi}) = x_i + y_i P_{SRi} \tag{22}$$

where  $x_i$  and  $y_i$  are the spinning reserve cost of the  $i^{th}$  thermal generator.

#### 4. Optimization method

The problem defined above needs a multi-objective optimization approach to be solved. Then, this section describes the proposed algorithm. First, we develop the operating process then, we present the flow chart of the proposed optimization approach.

#### 4.1. Multiobjective optimization

A general multiobjective optimization problem can be mathematically expressed as follows [13] :

$$\begin{array}{ll} \text{Minimize} & \mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_{N_{\text{obj}}}(\mathbf{x})] \\ \text{Subject to} & g_k(\mathbf{x}) \leq 0, k = 1, \dots, N_c, \end{array} \quad (23)$$

where  $\mathbf{x} = [x_1, x_2, \dots, x_D]^T$  with  $x_j$  can be either real, integer or boolean values, and  $D$  is the research space dimension.  $f_r(\cdot)$  are the  $N_{\text{obj}}$  objective functions and  $g_k(\cdot)$  are the  $N_c$  constraint functions of the problem.

The family of optimal solutions of this MOP is composed of all those potential solutions such that the components of the corresponding objective vectors whose elements cannot be simultaneously improved. This is known as the concept of Pareto optimality. In a minimization problem, Pareto dominance and Pareto optimality are defined as follows [14] :

**Definition 1** (Pareto dominance). *A given vector  $\mathbf{x} = [x_1, x_2, \dots, x_D]$  is said to dominate  $\mathbf{y} = [y_1, y_2, \dots, y_D]$  if and only if  $\forall j \in \{1, 2, \dots, D\}, x_j \leq y_j$  and  $\exists j_0 \in \{1, 2, \dots, D\}, x_{j_0} < y_{j_0}$ .*

**Definition 2** (Pareto optimality). *For a general MOP, a given solution  $\mathbf{x}^* \in \mathcal{F}$ , where  $\mathcal{F}$  is the feasible solution space, is Pareto optimal if and only if there is no  $\mathbf{x} \in \mathcal{F}$  that dominates  $\mathbf{x}^*$ .*

#### 4.2. Fast and elitist multiobjective genetic algorithm

Fast and elitist multiobjective genetic algorithm (NSGA-II) is the second version of NSGA which improves this later to overcome the computation complexity and the non-elitist characteristic of solutions [15]. NSGA-II method starts by initializing the population and assigning to each point the appropriate rank. Thereafter, reproduction operators such as tournament selection, recombination and mutation are used to create the offspring population. Then, the two populations parent and offspring are combined and sorted following the comparison operators mentioned above. More details and complexity study of NSGA-II are given in [15]. The basic operations of NSGA-II are as follows:

- *Fast Non-dominated Sorting* which is based on two entities. The first one is the calculation of the number of solutions dominating each solution in the current population. This number determine the rank of each solution. The second entity is the set of solutions that a solution dominates. The sorting process of a population  $P$  is described in the algorithm 1.
- *Density estimation (crowding distance)*: presents the density of solutions surrounding a particular point in Pareto front. It is the average distance of two points on either side of this point along each of the objectives.
- *Crowded-Comparison Operator*: which compares two solutions on the basis of both the rank and crowded distance. The better solution is this with smaller rank. In the case of rank equality, the saved solution is this with smaller crowded distance.

### 5. Simulation and results

Hereafter, we use the proposed optimization algorithm to solve the power dispatch problem in the case of a IEEE 30-bus test network. This network which includes 30 buses, 6 thermal generators and 41 transmission lines [16]. Table 1 presents the fuel cost and emission function coefficients. The grid data and the buses loads on a 100MVA base of the test system are given in [16]. In order to show performances f the proposed approach, two cases are studied. In the first one, we consider the classical EED with and without considering the spinning reserve cost and which considers the TUs only. However, in the second case, the two first TUs generators are replaced by, the first one, wind power generator and the second one by PV cell generator of the same capacity 50MW and 60MW, respectively. The CDF of both wind and PV generators are presented in Figs. 1a and 1b, respectively. Figs. 2a and 2b shows the Pareto fronts with taking into account the spinning reserve cost in the second one. Comparing these curves with the obtained one in the case of considering the expected values of wind and PV generators as shown in Fig. 3, it is well observed that

**Algorithm 1** Fast non dominated sort

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1: for  $p \in P$  do
2:    $S_p = \emptyset$            {  $S_p$  is the set of solutions that the solution  $p$  dominates.}
3:    $n_p = 0$              {  $n_p$  is the domination count}
4:   for  $q \in P$  do
5:     if  $p < q$  then
6:        $S_p = S_p \cup \{p\}$        {  $p$  dominates  $q$  }
7:     else
8:        $n_p = n_p + 1$ 
9:     end if
10:    if  $n_p = 0$  then
11:       $p_{rank} = 1$ 
12:       $F_I = F_I \cup \{p\}$ 
13:    end if
14:  end for
15: end for
16:  $i = 1$                  {Initialize the front counter}
17: while  $F_i \neq \emptyset$  do
18:    $Q = \emptyset$ 
19:   for  $q \in S_p$  do
20:      $n_q = n_q - 1$ 
21:     if  $n_q = 0$  then
22:        $q_{rank} = 1$ 
23:        $Q = Q \cup \{q\}$ 
24:     end if
25:   end for
26:    $i = i + 1$ 
27: end while

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the cost and emission functions get better values. But, it is important to note that this curve reflect only the expected values of RES generations. Three particular solutions are chosen, especially extreme points and a middle one, in order to show the CDF of all possible total costs, as illustrated in 4b, which are elaborated based on the CDF of total required reserve cost as shown in Fig. 4a.

	$G_1$	$G_2$	$G_3$	$G_4$	$G_5$	$G_6$	
Cost coef	a	10	10	20	10	20	10
	b	200	150	180	100	180	150
	c	100	120	40	60	40	100
	x	30	35	25	30	25	30
	y	300	190	320	310	320	310
Emission coef	$\alpha$	4.091	2.543	4.258	5.326	4.258	6.131
	$\beta$	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
	$\gamma$	6.490	5.638	4.586	3.380	4.586	5.151
	$\psi$	2.0e-4	5.0e-4	1.0E-6	2.0E-3	1.0e-6	1.0e-5
	$\lambda$	2.857	3.333	8.000	2.000	8.000	6.667
Limits	$P_{min}$	5	5	5	5	5	5
	$P_{max}$	50	60	100	120	100	60

Table 1: Cost and emission coefficients of IEEE-30 generators

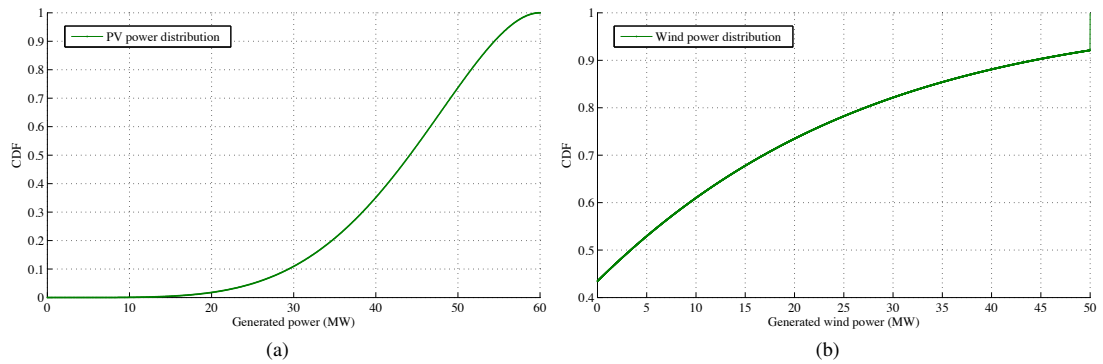


Fig. 1: Cumulative distribution of (a) PV cell generation and (b) wind power generation

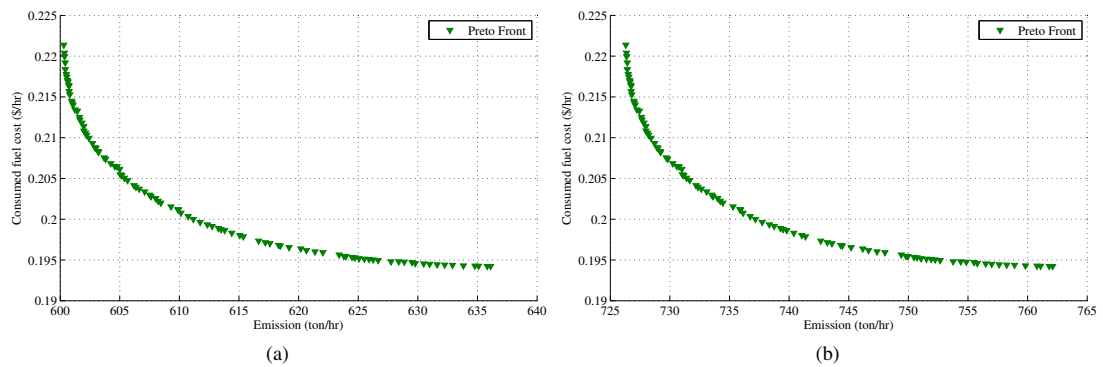


Fig. 2: Economic emission dispatch solutions in the case required reserve cost (a) not considered and (b) considered

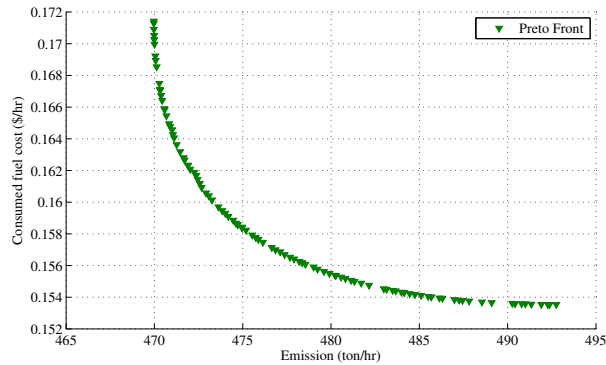


Fig. 3: Economic emission dispatch solutions considering expected values of RESs generations

## 6. Conclusion

This paper extends the classical multiobjective economic emission dispatch of a power system with only thermal units production to a probabilistic EED of a system with renewable energy sources. The proposed approach is based on determining the cumulative distribution function (CDF) of the required reserve in order to find the CDF of all the problem random variables and especially to determine the CDF of total operating cost. The obtained results show

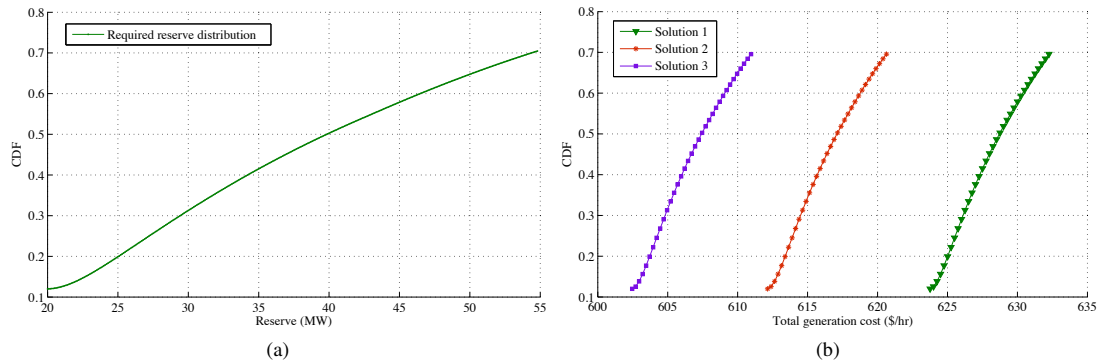


Fig. 4: Cumulative distribution function of (a) contributing reserve and of (b) three particular solutions

clearly the importance of the obtained performances allowing a great dispatch of the future power system generation. This study can be extended to analyze and to treat the probabilistic EED of power system with more RES diversity such as biomass, CSP with and without storage, geothermal, solar thermal and heliostat. And also in the case of combined heat and power.

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